WHY IS ALGEBRA IMPORTANT TO LEARN?

(Teachers, this one's for your students!)

BY ZALMAN USISKIN

We have all heard or read eloquent expositions on the value of studying history. Likewise, most people can quickly summon up a compelling case for the importance of good writing skills, the benefits of being exposed to quality literature and art, the need to understand basic scientific concepts, and the indispensable requirement to master the tools of everyday arithmetic.

But when it comes to providing the rationale for learning mathematics beyond arithmetic—namely, algebra, geometry, and their extensions—most people are likely to mumble something about needing it for college. Indeed, just this past year, a book questioning the importance of math education in general and arguing that almost no one really needs algebra received considerable attention and quite favorable reception in the popular press. And, more importantly, every day across this country countless numbers of students sit in algebra or geometry classes wondering (as did generations before them) why they are studying linear (not to mention quadratic) equations or a method for finding the volume of a cylinder. "What's the point? I'll never use any of this stuff," they say.

They deserve an answer. And while it is true that the newer textbooks do a better job than the older ones did of showing the practical applications of algebra and geometry, most do so in a scattered, piecemeal fashion. For some students, this poses no problem. They love math, they're good at it, they never question its value or appeal. But there are many others who need the whys and wherefores set forth in a convincing manner. And since more and more school districts are replacing dumbed-down "consumer math" courses with a requirement that all students take algebra and geometry, we thought the time was ripe to pull together, for a student audience, the case for studying post-arithmetic math. We convinced Zalman Usiskin, the director of the widely acclaimed University of Chicago School Mathematics Project, to take up the question of algebra in this issue. Perhaps in a future issue, we'll ask him to do the same for geometry. He wrote this essay for your students, and we hope you will share it with them.

—Editor

MOST PEOPLE realize that they need to know arithmetic. Whole numbers, fractions, decimals, and percents are everywhere. You need them to do any work with money, to deal accurately with measurements, to understand probability, to follow the results of election polls or sports or lotteries, and a wide range of other things. Numbers are everywhere. Just pick up a newspaper or magazine, open to any page at random, and count the numbers on it. You may be surprised at how many there are.

Algebra seems different. Scan the same newspaper and you are not likely to see any algebraic formulas. The need to know algebra isn't as obvious.

Furthermore, we all know adults who live productive lives without ever using algebra. Even adults who studied the subject as youngsters may not have studied its practical applications; hence it is difficult for them to see—let alone explain to others—what value they derived from it.

It is true that the value of algebra is not as obvious as the value of arithmetic. In actuality, however, its usefulness is all around us. But for those who don't know where or how to look, it is often hidden. It is well worth the effort to dig a little deeper to uncover the many ways this discipline is at work in the world and why mastering it greatly enriches our lives. In this essay, we hope to do some of that digging.

TO SAY "You need algebra for college," or "You won't do well on the SAT or ACT without it" are true statements, but they don't tell us very much. *Why* is algebra

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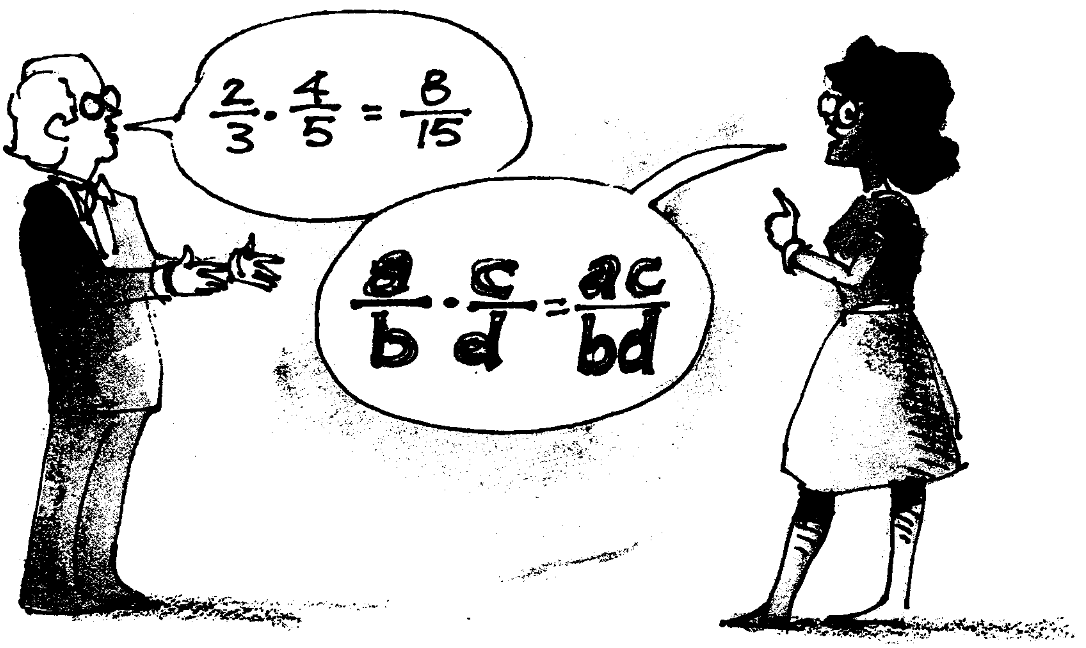


ILLUSTRATED BY ROBERT BARKIN

considered so important that it has become a requirement for entry to virtually every college? And why are more and more school districts requiring all students to study algebra, including those who may not be college-bound or who haven't yet made up their minds about their futures? In the pages that follow, we offer some answers to these questions. Here are some general reasons:

Without a knowledge of algebra,

- you are kept from doing many jobs or even entering programs that will get you a job;
- you lose control over parts of your life and must rely on others to do things for you;
- you are more likely to make unwise decisions, financial and otherwise; and
- you will not be able to understand many ideas discussed in chemistry, physics, the earth sciences, economics, business, psychology, and many other areas.

In these matters, algebra has much in common with reading, writing, and arithmetic: Lack of knowledge limits your opportunities. More specifically, what follows are the characteristics of algebra that cause it to be so important and some of the many things you could not do at all—or not do as easily—without it.

Algebra is the language of generalization. If you do something once, you probably don't need algebra. But if you are doing a process again and again, algebra provides a very simple language for describing what you are doing. Algebra is the language through which we describe patterns. For instance, here is the rule for multiplication of fractions, written in English words:

To multiply two fractions, multiply their numerators to get the numerator of the product, and then multiply their denominators to get the denominator of the product.

As an example,

$$\frac{2}{3} \bullet \frac{4}{5} = \frac{8}{15}$$

Here is the same rule, written in the language of algebra:

$$\frac{a}{b} \bullet \frac{c}{d} = \frac{ac}{bd}$$

Not only is the algebra much shorter, but it looks like the arithmetic!

Some general patterns are formulas. Formulas state one quantity in terms of another. There are formulas in every walk of life. For instance, there is a formula for finding Celsius temperatures from Fahrenheit temperatures

$$F = \frac{9}{5}C + 32,$$

and vice-versa

$$C = \frac{5}{9}(F - 32).$$

There are formulas for area that come in handy if you are looking for a place to live and want to know how much room you will have, or if you are sewing clothes and want to determine the amount of material you'll need. There are formulas for perimeter that tell how much fencing you might need for a field, or how much ribbon to tie a package. There are all sorts of formulas in sports, and they can help you calculate earned-run average in baseball (not hard), or rate a quarterback in football (rather complicated), or determine the probability of a particular player making two free throws in a row in a basketball game (easy but not certain). Income tax, discounts, sales tax, and virtually every money matter involve applying some formula. You can get along without the formulas—many people do—but you are less likely to be fooled by someone if you yourself can deal with the formulas.

Some patterns are not so simple. For instance, the formula

$$W = d + 2m + \left\lfloor \frac{3(m+1)}{5} \right\rfloor + y + \left\lfloor \frac{y}{4} \right\rfloor - \left\lfloor \frac{y}{100} \right\rfloor + \left\lfloor \frac{y}{400} \right\rfloor + 2$$

tells you on which day of the week a particular date will fall, even years into the future, where

d = the day of the month of the given date
m = the number of the month in the year, with January and February regarded as the 13th and 14th months of the previous year. The other months are numbered 3 to 12 as usual.
y = the year.

(You may not be familiar with the $\lfloor \ \rfloor$ symbol—brackets without the top corners—which means round the number inside it down to the nearest integer. For instance,

$$\lfloor 19.23 \rfloor = 19; \lfloor 497 \rfloor = 497; \text{and } \left\lfloor \frac{42}{5} \right\rfloor = 8.)$$

Once W is computed, divide W by 7 and the remainder is the day of the week, with Saturday = 0, Sunday = 1, and so on, with Friday = 6. You might try the formula on today's date to see that it works. It even accounts for the fact that the year 2000 will *not* be a leap year.

Algebra enables a person to answer *all* the questions of a particular type at one time. Suppose you are thinking of borrowing some money. Typically, money borrowed for a car or a house or other large purchase is paid back monthly, but the interest rate is given as a yearly rate. The key is how much money per month you must pay back, and this is not easy to calculate. Even if you use tables, you may not find tables with the rate you will be charged or the amount you wish to borrow. But there is a formula that can calculate the monthly payment P if an amount A is borrowed for m months at an r% annual rate.

$$P = Ax^m \frac{x-1}{x^m - 1}, \text{where } x = 1 + \frac{r}{1200}$$

With a scientific or graphics calculator, you can use this formula even if the amounts are strange. With this formula, you can calculate that a 4-year loan of $8,500 to buy a car, at an 11.25% annual rate, requires a monthly payment of $220.72.

Not all formulas are so complicated. Suppose you want to calculate the miles per gallon that a car is getting. If the car's gas tank is filled at 25,000 miles and then again at 25,400 miles, and 15.3 gallons of gas have been used by the time of the second fillup, you could subtract to find the number of miles, and then divide by 15.3 to find the miles per gallon, in this case about 26 mpg. You can do a single problem like this just using arithmetic. But if you had to answer a lot of questions of this type, you would want to have a general procedure. If the first fillup is at f miles and the second at s miles, and you fill up with g gallons of gas the second time, then the miles per gallon is

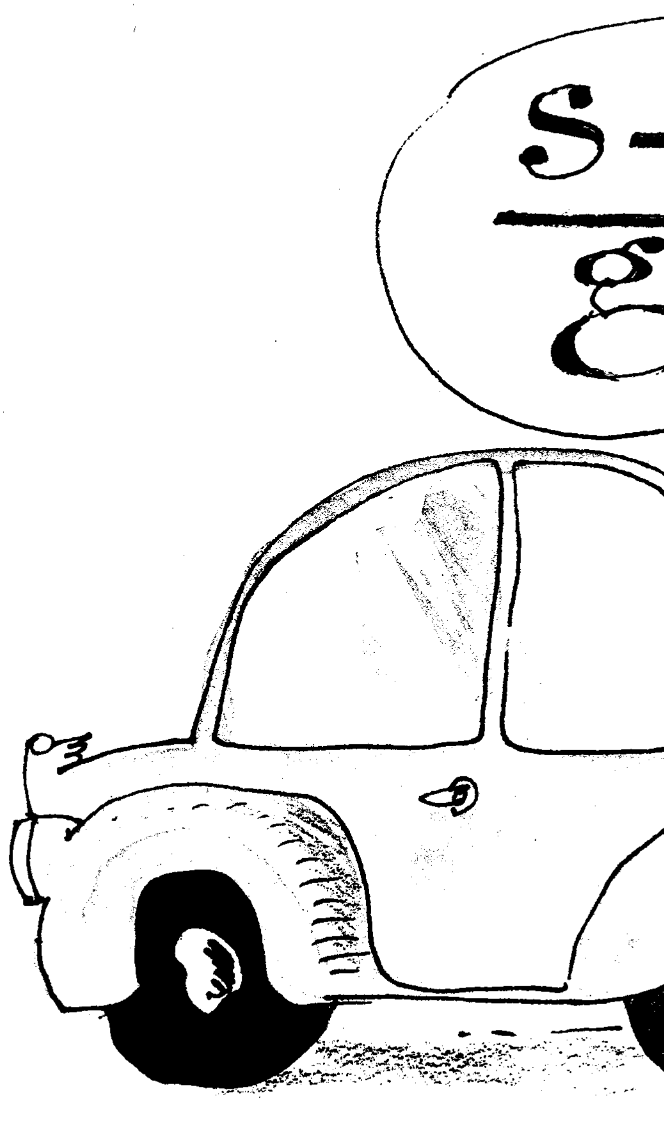
$$\frac{s-f}{g}.$$

It's a great deal easier to substitute into a formula than to have to figure out how to do the question each time.

Computer programs use the language of algebra because they require a language that will work for a variety of inputs. Formulas are also the backbone of today's spreadsheets. It used to be that only accountants used spreadsheets, to keep records of financial matters. Now spreadsheets are used for all sorts of recordkeeping.

Algebra is the language of relationships between quantities. What happens to various aspects of your health as you grow older, or as your weight changes, or as you eat different foods or take different medicines? What will it cost to produce an article of clothing if more are made in a factory or if a different material is used? How will your budget be affected if you change your spending habits? How is the energy use of the world affected by its population? How does a tree grow over time? How is Celsius (Centigrade) temperature related to Fahrenheit temperature?

These kinds of questions, in which one quantity depends on others, are the basis of functions. When people do not know algebra, functions are often described by long tables. Many functions can be pictured by graphs. But the algebraic description is the shortest and often the one easiest to use; moreover, it can give information that the other descriptions do not contain. Functions are studied when one wishes to find out what happens to a quantity in the long run. For instance, will the population of a particular animal in a region stabilize, increase, or decrease? Functions are also important for determining when a quantity will reach its highest or lowest value. These are two of the types of questions that form the



basis for calculus, and they require that someone know algebra first.

Algebra is a language for solving certain kinds of numerical problems. It used to be common to see problems involving age, motion, coins, work, and mixtures in algebra books. Now algebra books are just as likely to have problems that involve everyday situations. How much of a particular food can you eat and stay within a particular diet? How much can you spend and still be within budget? When there is a formula relating two quantities, if you know one, you can find the other. And today's technology has made it possible to work with far more complicated formulas than one could deal with even a generation ago.

Some Algebraic Topics and a Few of Their Applications

Linear equations: Anything that changes at a constant rate gives rise to a linear equation of the form $T = Ax + By$ (and sometimes many other variables). This applies to the total cost of items when each item costs the same, total calories or vitamins or minerals consumed in food, total amounts of material in producing objects, total electricity or other energy costs on household bills, costs of renting a car, and the cost of a long-distance call. For instance, algebra can indicate from a phone bill what the initial cost and additional cost per minute was on a long-distance call; it can also tell how long a person could talk for any particular number of dollars.

Slope: Any quantity that changes must change at some rate. The rate of change is often very important. In business, how much the cost of producing changes when one more item is produced is called the marginal cost. How fast a car changes speed is called its acceleration or deceleration. How fast your money increases or decreases can affect your financial status. How fast the unemployment rate goes up or down can affect inflation. The algebraic idea behind all these notions is slope, and is studied usually in a first course.

Small parts of smooth curves can often be approximated well by lines. These lines can be described by a linear equation. For instance, the world record in the mile run for men (the most famous track-and-field record) since 1875 is quite well approximated by the equation

$$t = -.346Y + 914.156,$$

where t is the time in seconds and Y is the year. (For 1995 this gives a value of 3 minutes 43.89 seconds; the record is currently 3 minutes 44.39.) The slope in this equation, which can be obtained automatically using most graphics calculators available for schools today, indicates that the record has been decreasing by about 0.346 seconds a year on average, which should indicate to runners and their coaches that there is still a way to go before the limit of human capability is reached.

Exponents: The area of a square with side s is s^2; the volume of a cube with edge e is e^3. These applications were known to mathematicians more than 2,000 years ago, and connect algebra with geometry. In the Middle Ages, mathematicians began to study games of chance, and out of these games came the notions of probability and odds, which today require a knowledge of exponents. If something grows at a constant percent of itself, the growth is called exponential. Exponential growth is found in all the largest money matters that affect people: the compound interest by which savings institutions calculate interest; car, mortgage, and credit card payments for anything bought on time; retirement funds and life insurance and other types of annuities. The same mathematical concept underlies population growth and helps in predicting not only human population sizes but also animal and plant populations and sizes of tumors and other aspects of disease. So people interested in finance, business, accounting, environmental issues, and medicine use these ideas often.

Quadratics: It was Isaac Newton who discovered that any object that is affected by gravity will travel on a path described by a quadratic expression of the form $Ax^2 + Bxy + Cy^2 Dx + Ey + F$. This applies to stars and planets and comets and meteors and moons; to rockets and paths of bullets; and to paths of baseballs hit and basketballs thrown and any other objects thrown or shot or travelling without additional propellants. So this idea is used in physics, in space science, in the armed services, in sports, and in hobbies or other activities that involve throwing or propelling objects.

Quadratic expressions also describe counting situations that involve pairs, such as the number of games necessary for teams in a league to play each other, or the number of connections needed between two cities. They are involved in the fundamental formula for the distance between two points, in formulas for momentum and force and pressure and other physical quantities.

Logarithms: Logarithms are seldom used today for computation; calculators and computers have taken that drudgery away. Their uses are found now most commonly in understanding scales that need to cover quantities of vastly different magnitudes, such as the Richter scale for earthquakes, the Ph scale used to determine acidity or alkalinity in chemistry, the star magnitude scale used in astronomy, and in many statistical scales. Logarithms also are needed to solve problems involving exponents; for instance, the solution to an equation like $(1.06)^x = 2$ (How long will it take for money invested at 6% annually to double?) is most easily described using logarithms

$$\left(x = \frac{\log 2}{\log 1.06} \approx 12 \text{ years}\right).$$

Permutations and Combinations: These ideas, now usually found in more advanced high school algebra courses, began as a result of studies of gambling. Today they are used to determine the odds of winning lotteries or card games or other games of chance. This is the reason that people who know the mathematics are less likely than other people to gamble unwisely. Permutations and combinations help to determine the number of people or objects needed to be involved in political and other opinion polls, TV ratings, and product testing, and they also help in determining the accuracy of the poll or test. Consequently, they are important tools for jobs that require the manufacture of high-quality products.

A Sampling of Algebra Problems

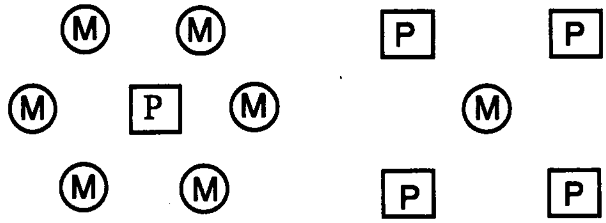
Out of the huge number of situations and questions that can be solved by algebra and are accessible to most high school students, we have picked ten. In parentheses following the examples are the algebraic ideas needed for the solution. (The answers may be found on p.46.)

1. *The World Almanac and Book of Facts* 1995 lists 59 major earthquakes from 1940 to 1994. Here are their frequencies by season of the year: Autumn, 14; Winter, 14; Spring, 11; Summer, 20. Use statistics to determine whether these frequencies support a view that more earthquakes occur at certain times of the year than at others, or if differences like these occur commonly by mere chance. (quadratic expressions)

2. To estimate the number N of bricks needed in a wall, some bricklayers use the formula N = 7LH, where L and H are the length and height of the wall in feet. About how many bricks would a bricklayer need for a wall 8.5 feet high and 24.5 feet long? (formulas)

3. Nellie and Joe plan to save money for an around-the-world trip when they retire 10 years from now. Nellie plans to save $1,000 per year for the first 5 years and then stop making deposits. Joe plans to wait 5 years to begin saving but then to save $1,200 a year for 5 years. If they deposit their savings at the end of the year into accounts yielding 6% a year, how much will each person have after 10 years? (polynomials)

4. A marching band has 52 musicians and 24 members of the pompom squad. One of the drills involves hexagons and squares like those shown below. Can these formations be executed in such a way that no one is "left over"? If so, how many hexagons and how many squares can be made? (systems of linear equations)



Hexagon with pompom person in center — Square with band member in center

5. Architects designing auditoriums use the fact that sound intensity is inversely proportional to the square of the distance from the sound source. A person moves from one seat to another 4 times the distance from the source. How is the intensity of the sound affected? (functions of variation)

6. One rental car company charges $21.95 per day plus 41¢ a mile. Another charges a flat rate of $39.95 per day with unlimited mileage. For what driving distances should you pick the second company over the first? (linear equations)

7. A batter hit a baseball when it was 3 feet off the ground. It passed 4 feet above the 6-foot-tall pitcher 60 feet away. It was caught by an outfielder 300 feet away, 5 feet off the ground. How far from the batter did the baseball reach its maximum height, and what was that height? (quadratics and systems)

8. Rock music played at 120 decibels is how many times as intense as music played at 105 decibels? (logarithms)

9. In 1992, the population of Mexico was 92.4 million and growing at a rate of about 2.4% annually. If this growth rate continues, in about what year will the population of Mexico reach 100 million? (exponentials and logarithms)

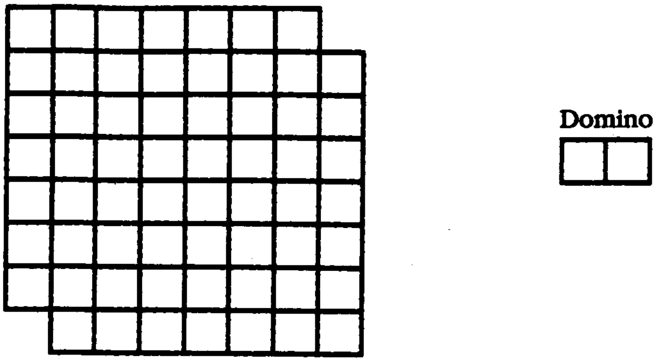
10. POWERBALL™ is a lottery played in 15 states. To play, an individual picks five numbers from 1 to 45 and a sixth "powerball" number from 1 to 45. To win the jackpot, which is always at least $2 million, you must match all 5 numbers (in any order) plus the powerball. What is the probability of winning the POWERBALL™ jackpot? (combinations)

The Importance of Deduction

It is often said that you can be certain of the answer to a mathematics problem. There is a sense in which this is true. If you agree with the assumptions in the problem, and if valid reasoning has been used, then you must agree with the solution or solutions. This property of the solving of mathematical problems is a result of a basic underlying aspect of mathematical thinking—the *deductive process*, or *deduction* for short.

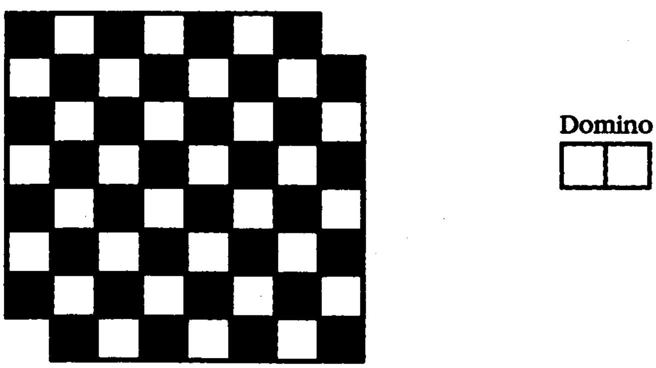
Deduction is the logical process by which the truth of one statement follows from the truth of others. For mathematicians, only results that have been deduced from agreed-upon statements using valid arguments of deduction can be thought of as *true*. Deduction is used throughout mathematics.

Deduction is a difficult criterion for the establishment of truth because it means we cannot say something is true merely because we have lots of examples. For instance, below is pictured an 8-by-8 grid, but with two opposite corners removed.



Can the grid be covered with 31 dominoes that each cover two squares? If you try to actually do this, you will find that no matter how you fit the dominoes together they will not cover the squares. You can cover 60 of the squares but the 2 that remain will not be next to each other. But all the unsuccessful attempts in the world do not prove that someone could not come up with a way to cover the grid with 31 dominoes. Only deduction can do that.

Here is a proof. First, color the squares like a checkerboard. This does not change the problem. If dominoes can cover the original grid, they can cover this checkerboard, and vice versa.



Now each domino always covers two adjacent squares, one black and one white. The 31 dominoes must thus cover 31 black squares and 31 white squares. But the opposite corners of an 8-by-8 checkerboard are the same color, so by removing the opposite corners, we are left with 30 of one color (white above) and 32 of the other (black above). No matter how 30 of the 31 dominoes are placed, there will be two black squares without dominoes on them, and the remaining domino cannot cover both. The deductive argument *proves* that any attempt to cover this cut-off checkerboard with 31 dominoes will be futile.

Deduction can also indicate whether other checkerboards can be covered with these dominoes. For instance, a 5-by-5 grid cannot be covered with dominoes because there are 25 squares, an odd number. But a 9-by-8 grid can be covered whether or not its two opposite corners are removed.

The covering of a grid with dominoes is a type of problem called a *partition* or *covering* problem. It is not so unrealistic a problem. Any metalworker who needs to cut items from a sheet of metal, or a clothesmaker who needs to cut pieces of cloth from a bolt may be faced with this kind of question: Can the items be cut with no pieces left over? Boxes can be formed by cutting off corners and folding the remaining pieces up. It saves money if no pieces are left over. Thus the same process of deduction that is applied to grids and dominoes may be useful in many other situations.

The process of deduction is also found outside of mathematics. It is used by lawyers and union negotiators and by anyone else trying to make a convincing argument or build their case. It is thought by many people to be the purest form of reasoning, and it comprises another of the reasons that many people believe mathematics is important for all students to study.

The Wonder of It All

You would not be able to understand this article if you could not read. Even though most students learn to read in elementary school, we teach the reading of literature in high school and throughout college. The reason for doing this is not just that reading literature is needed for jobs, or because literature may help in solving everyday problems. It is also because many people find good books enjoyable, relaxing, exciting. Reading is fun!

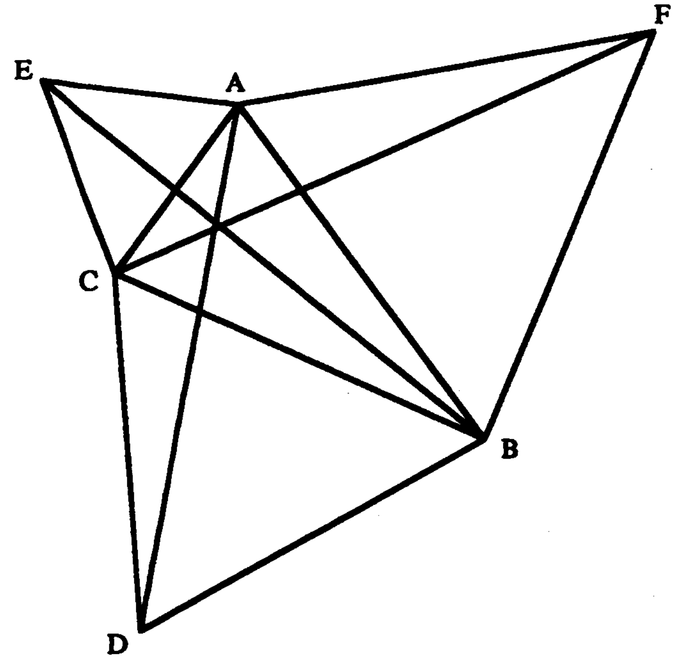
It is no different with mathematics. Not everyone studies mathematics because of its usefulness. Many people study mathematics because they enjoy its puzzles, which are like mysteries in literature. Many people study mathematics because they like the way that it all fits into a structure, somewhat like the way a book or a piece of music of high quality can be analyzed for its structure. Many people study mathematics because of its beauty.

Some of the beauty of mathematics comes from its wonder. Just how was that result discovered? How is it proved? For instance, add up the digits of a whole number. Then the number is divisible by 9 if and only if the sum of digits is divisible by 9. For example:

- 4,374 is divisible by 9 because the sum 4+3+7+4 is 18, and that is divisible by 9;
- 91,604 is not divisible by 9 because the sum of its digits is 20, and that is not divisible by 9.

We know this property of whole numbers written in our base-10 decimal system is true because it has been proved. The proof requires only the algebra studied in high school.

Geometry, too, is famous for its wonderful relationships. Here is one with a surprising use. Start with any triangle. We call ours ABC. On each of the sides of triangle ABC, construct an equilateral triangle. In the figure, tri-



angles BCD, ACE, and ABF are equilateral.

Now connect AD, BE, and CF. It looks as if these three lines intersect at the same point. In fact, for any triangle, it can be proved that these lines do intersect at the same point. That point is called the Fermat point of the triangle, named after Pierre Fermat, a 17th century French lawyer and mathematician. When the original triangle has no angle as great as 120°, the Fermat point has the property that the sum of its distances to A, B, and C is smaller than that sum for any other point. So, if three towns wish to have an airport, and access roads will have to be built from each town to the airport, or if there is to be a cable network connecting the towns and cables need to be laid from the towns, the Fermat point may be the cheapest spot for the airport or central switching location.

The result is not only useful, it is beautiful. Notice that the six angles formed by the lines at the Fermat point seem to be equal. In fact, each is a 60° angle. It is surprising and wonderful that even when one begins with a triangle that has no symmetry, one winds up with a point in the middle with as much symmetry as one could have. Many people study mathematics for the enjoyment that seeing, discovering, and proving such results brings.

And, finally, there is mystery. Sometimes it is not known whether a statement is true or not. Recall that a prime number is a whole number greater than 1 that is divisible only by itself and 1. The primes smaller than 100 are 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, and 97. More than two thousand years ago it was proved that there are infinitely many primes. The proof is ingenious: Suppose you had a list of all primes. Now multiply all those primes together and add 1. The result cannot be divisible by any of the primes you've multiplied, because two consecutive whole numbers are not divisible by the same number larger than 1. So the result must either be another prime or divisible by a prime larger than the ones on your list. This shows that no finite list of primes has them all. So the number of primes must be infinite.

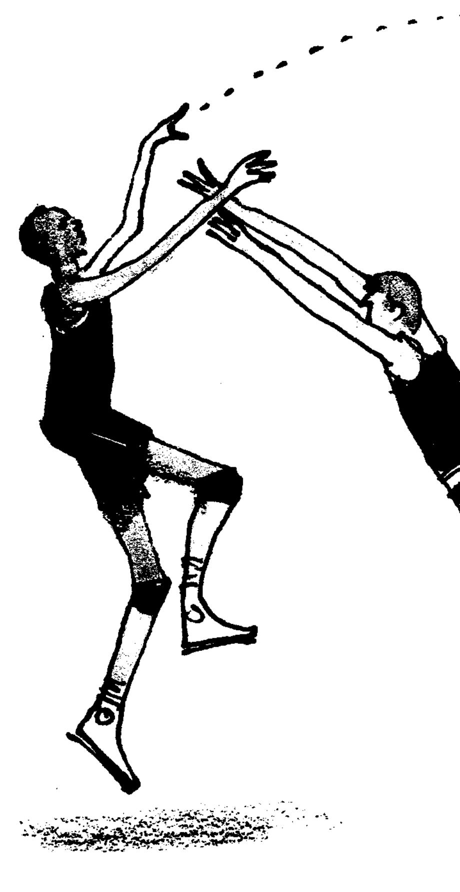
Even with their long history, there remain many things not known about primes. One mystery involves pairs of primes that differ by 2. These are called twin prime pairs. The twin prime pairs less than 100 are: 3 and 5, 5 and 7, 11 and 13, 17 and 19, 29 and 31, 41 and 43, 59 and 61, 71 and 73. Are there infinitely many twin prime pairs? No one knows. About all that we know is that there are very large pairs of twin primes. In 1988, a pair of twin primes with 2,259 digits each was discovered!

Until this century, most people thought of primes as a part of mathematics that had no applications. But in the last thirty years, large prime numbers (with perhaps 100 digits) have become an essential part of the security systems used to protect bank accounts, confidential information, and national security interests. This is one reason why the largest employer of Ph.D. mathematicians in the United States is the National Security Agency outside Washington, D.C.

If Mathematics Beyond Arithmetic Is So Important, Why Have Many Adults Been Able To Get Along Without It?

It is common for adults today to speak of algebra and geometry and other mathematics beyond arithmetic as if they are important only to a few people. For instance, as we were preparing this essay, an article in the sports section of a Chicago newspaper began with this sentence: "When the regular season begins in three weeks, Friday night's Bulls preseason opener will become about as significant as algebra formulas learned in high school."

Doesn't this sportswriter know that many of the sports statistics that are printed in his newspaper are calculat-



ed using algebraic formulas? For instance,

$$A = \frac{T}{G}$$

is the formula for the average number of points (A) a player scores per game if the player has scored T total points in G games. The winning percentage of a team is,

$$\frac{W}{W+L},$$

where W is the number of wins and L the number of losses.

It is likely that the writer does know these formulas, for most newspaper staff are college graduates. However, these writers and many other adults avoid the formulas when they can. They are like people who go to a foreign country but do not know enough of the language to converse with native speakers in that country. If you visit Mexico but do not know Spanish, you can get along, but you will never appreciate the richness of the culture, and you will not be able to learn as much as you could if you knew Spanish. You will be forced to depend on signs that have been translated into English. And perhaps most significantly, you will not even know what you missed.

And so it is with algebra. You can live without it, but you will not appreciate as much of what is going on around you. You might not be eligible for the job you would like to have or the training program or courses you would like to take. You will not be able to participate fully in our technological society. You will be more likely to make unwise decisions, and you will find yourself with less control over your life than others who have this knowledge. You will live in the same world, but you will not see or understand as much of its beauty, structure, and mystery. And, quite possibly, you will not have as much fun! □

Bibliography

All examples and problems in this article are found in three books of the University of Chicago School Mathematics Project, published by ScottForesman: *Algebra, Advanced Algebra, and Geometry.*

For students who are interested in a particular field, it is useful to consult specific texts relating mathematics to that field. Look for titles that begin *Mathematics for ... (biologists, nurses, electricians, etc.)* or in texts for that field.

Here are some valuable general references:

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